ESTIMATING FRACTURE DENSITY FROM A LINEAR OR AREAL SURVEY

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ABSTRACT

Fracture density is an important parameter of a fracture network. There are several definitions of fracture density and we are concerned here with the average fracture surface per unit volume. When fractures are not distributed equally among all directions, this parameter can be considered as a function of fracture pole. It can be evaluated from areal and/or linear surveys. This evaluation must account for the geometric bias due to the fact that fractures oblique to the scanline or surface are less easily observed than fractures orthogonal to it. A well-known technique to achieve this is the Terzaghi correction. We propose here an improved correction when survey outcrops and/or boreholes with different orientations are available.

INTRODUCTION

Fractures have a large impact on the recovery of oil, the safety of underground storage of CO_2 or nuclear waste, the assessment of high-enthalpy geothermal installations, the safety of mining exploitations, the efficiency of in-situ leaching operations or of extraction by block-caving. Fractures is a generic term which includes a variety of objects occurring at all scales: faults (with a clear lateral displacement of one surface with respect to the other), joints, veins, etc. The methodology which follows is not specific to a type of fractures but in applications shall be applied separately to the various fracture types considered.

Usually we study separately the fracture network and the single fracture. In the latter case we consider the exact shape of a fracture: topography of its two walls, roughness, geometry and continuity of the void space, filling material, traces of movement, reaction to mechanical stress or water injection. In the former case a fracture is considered as a planar object and stress is laid on the statistical and geometric distribution of the objects: fracture density, distribution of fracture orientation and fracture size, relationships between the various fracture sets, etc. (see, e.g., Chilès, 2005).

In practical applications we have to choose a sensible fracture network model and fit its parameters. Most models have a large number of parameters, because several fracture sets are present, each one with its own characteristics, not to speak of their interrelationships. Some of them allow a direct evaluation of their parameters, such as the regionalized disc-cluster model designed for granitic rocks (Chilès, 1988), and a hierarchical model designed for layered rocks (Chilès et al., 2000). Important parameters of nearly all models are fracture density and fracture size. The difficulty of their inference is that a fracture network is a 3D entity whereas observations are limited to 1D (boreholes) and 2D (outcrops, drift walls), and moreover very often to rather short stations, so that the data sets are subject to geometric bias, truncation, censoring, etc.

Many authors have shown how to infer the distribution of fracture traces from the sampling of outcrops, bench faces or drift walls, in particular in the framework of a maximum likelihood or Bayesian approach; see, among others, Baecher (1980), Laslett (1982), and Lantuéjoul et al. (2005). Once the shape of the fractures is known, the distribution of fracture size can be deduced from that of trace length, for example as shown by Warburton (1980) for disc-shaped fractures. We will focus here on the other parameter, fracture density, which is also of prime importance.

There are several definitions of fracture density (see Dershowitz and Herda, 1992), the main ones being the mean number of fracture centres per unit volume (λ) and the mean fractured surface per unit volume (μ). (Since we will not consider the fractures as three-dimensional objects – except by introducing attributes such as thickness or opening – we will not consider the definition of fracture density as the average fractured volume per unit volume.) When fractures are considered as finite objects, these parameters are related by

$$\mu = \lambda A \tag{1}$$

where A denotes the average fracture surface. If fractures are not distributed equally in all orientations, the above parameters shall be considered as functions of fracture pole orientation ω (the normal to the fracture plane).

There are many situations where the direct estimation of fracture density λ and fracture size is associated with a large uncertainty, whereas density μ can be estimated with more robustness. The problem originates in the stereological component of the problem, and also in the difficulty of identifying a fracture, as shown in Figure 1: what is observed as a series of three fractures on the outcrop can be considered as three distinct fractures, or as a single en-échelon fracture, or as a standard fracture whose observation is affected by erosion, vegetation, etc. According to the interpretation, we record either a single large fracture, or three short fractures. This has a large impact on fracture density λ and fracture size A but does not alter fracture density μ. It is thus safe to first estimate μ, which can be done in rather good conditions, even if only borehole data are available. The fracture size distribution will be inferred – or simply assumed if outcrops are not available – and fracture density λ, which is required by most models (e.g., Boolean models), will then be deduced from formula (1).

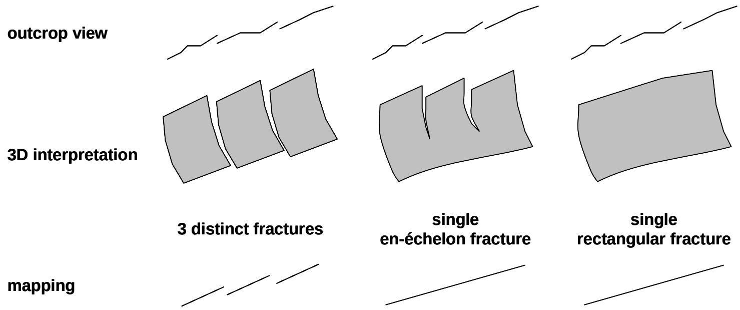

Figure 1. Three possible interpretations of the same outcrop view.

TERZAGHI CORRECTION

The main problem when estimating areal fracture density μ is that fractures oblique to the survey plane or line are less easily observed than fractures orthogonal to it. Figure 2 illustrates the problem in the two-dimensional case, for simplicity. This is the situation we have when all fractures are subvertical, which is common for joints in subhorizontal sedimentary formations. Let φ represent the acute angle between the scanline and the fractures of a set of infinite parallel fractures. The apparent fracture spacing along the scanline coincides with the true spacing if the fractures are orthogonal to the scanline, namely if $\varphi = \pi/2$, and is larger than the true spacing otherwise. The ratio of the apparent spacing to the true spacing is $1 / \sin \varphi$. This means that fractures at an acute angle φ with the scanline are underrepresented by a factor $\sin \varphi$.

In 2D, denoting the scanline direction by α and the fracture direction (azimuth) by θ, the factor is $| \sin(\theta - \alpha) |$ and therefore depends on the fracture set. The orientation of the fractures can also be represented by the direction $\omega = \pi/2 + \theta$ of their normal. The factor $| \sin(\theta - \alpha) |$ shall then be replaced by $| \cos(\omega - \alpha) |$.

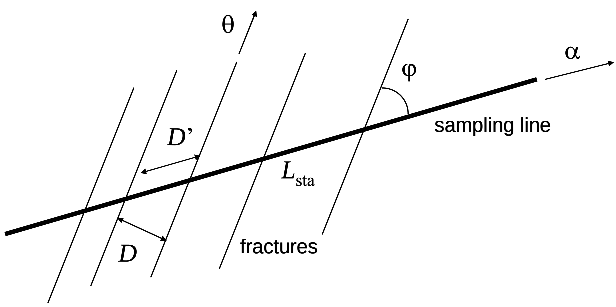

Figure 2. Apparent spacing D' associated with true spacing D when the fractures (direction θ) form an angle φ with the scanline (direction α).

In 3D the orientation of a fracture is defined by its pole, usually the unit vector ω normal to the fracture and directed towards the lower-hemisphere. The direction α of a scanline is also a unit vector. Fractures are perfectly observed when ω and α coincide; otherwise they are underrepresented by a factor equal to the cosine of the angle formed by the unit vectors ω and α, namely the absolute value of the inner product $<\omega, \alpha>$.

In the sequel we will characterize the fracture orientation by the fracture pole ω, which has the advantage of being defined in 2D as well as in 3D. To avoid being too formal, we will represent the factor by $|\cos(\omega - \alpha)|$, being understood that this means $|<\omega, \alpha>|$ when ω and α are unit vectors.

A means to compensate the underrepresentation of oblique fractures in the estimation of fracture density is to weight each fracture by $1 / |\cos(\omega - \alpha)|$:

$$\hat{\mu} = \frac{1}{L}\sum_{i=1}^{N}\frac{1}{|\cos(\omega_i - \alpha)|} \tag{2}$$

where L is the length of the scanline, α its direction, N the number of fractures, and ω_i the pole of fracture i. This is the Terzaghi correction (1965). The assumption of infinite fractures in fact plays no role in that result, nor the assumption of parallel fractures. This is why the correction is applied to each observed fracture.

Another way to obtain this result is to consider the scanline as a cylinder with length L, direction α, and a very thin circular section with area s, and to measure the fracture density μ in that cylinder. The intersection of fracture i with the survey cylinder is an ellipse with area $s_i = s / |\cos(\omega_i - \alpha)|$. The experimental fracture density is thus

$$\hat{\mu} = \frac{\sum_i s_i}{Ls} = \frac{1}{L}\sum_{i=1}^{N}\frac{1}{|\cos(\omega_i - \alpha)|}$$

An immediate generalization, when fractures are sampled along a line with a varying orientation or along several lines with total length L is

$$\hat{\mu} = \frac{1}{L}\sum_{i=1}^{N}\frac{1}{|\cos(\omega_i - \alpha_i)|} \tag{3}$$

where α_i is the local orientation of the scanline at the location of fracture i.

This estimator can be used with all the fractures (global fracture density), with those of a specific fracture set (fracture density of that set), or with the fractures of a specific direction ω. In the latter case, by varying ω, we have an estimator of the directional fracture density, which can be represented, for example, on a Schmidt diagram:

$$\hat{\mu}(d\omega) = \frac{1}{L}\sum_{i=1}^{N}\frac{1_{\omega_i \in d\omega}}{|\cos(\omega_i - \alpha_i)|} \tag{4}$$

In practice, Terzaghi correction gives a large weight to fractures subparallel to the scanline, and even an infinite weight to fractures exactly parallel to the scanline. Of course, such fractures should not be observed. But fractures are not perfect planes, the scanline is not perfectly linear, even locally, directional measurements are affected by some uncertainty, so that very large weights are often observed. The estimators (2) and (3) are thus not robust.

Mauldon and Mauldon (1997) generalize Terzaghi correction to the sampling by a borehole with nonzero diameter. In that case the correction always remains finite. Their result is very useful when we are interested in fractures with a size comparable with the diameter of the borehole. However, in the situation we envisage (fractures much larger than the borehole diameter) the correction can be very large.

Yow (1987) thus recommends not to take account of fractures that are too oblique to the scanline direction. The limit depends on the roughness of the outcrop, the shape of the fractures, the measurement accuracy. A limit of 15° is often used. This improves the robustness of the estimator but has the inconvenience of introducing some bias by discarding valid data. We therefore propose a variant of Terzaghi correction which represents a valuable improvement when outcrops or boreholes with different directions are available.

IMPROVED TERZAGHI CORRECTION

The principle of the method is to consider that, for the study of fractures with pole ω, a station with length L_{sta} and direction α provides neither more nor less information than a station orthogonal to the fractures and with length $L_{sta} \mid \cos(\omega - \alpha) \mid$. This leads to define an equivalent length, function of ω, as

$$L(\omega) = L_{sta} \mid \cos(\omega - \alpha) \mid$$

When several stations are available with lengths $L_1, \ldots, L_n$, and directions $\alpha_1, \ldots, \alpha_n$, respectively, the total equivalent length for fractures with direction ω is

$$L(\omega) = L_1 \mid \cos(\omega - \alpha_1) \mid + \ldots + L_n \mid \cos(\omega - \alpha_n) \mid$$

If $N(d\omega)$ fractures with pole in the solid angle $d\omega$ around ω have been observed in these n stations, the fracture density for polar direction ω is

$$\hat{\mu}(d\omega) = \frac{N(d\omega)}{L(\omega)} = \frac{\sum_{i=1}^{N} 1_{\omega_i \in d\omega}}{\sum_{j=1}^{n} L_j \mid \cos(\omega - \alpha_j) \mid} \qquad (5)$$

In practice, taking into consideration that the model (planar fractures, rectilinear stations) is an approximation to the reality and that the measurements are affected by measurement errors, in that definition $N(d\omega)$ and $L(\omega)$ are replaced by weighted averages (for example the average in a window of 15° centred on ω).

This approach allows a robust and unbiased estimation of directional fracture density while discarding no data, provided that the survey stations do not all have the same orientation. Otherwise, it amounts to Terzaghi correction (4).

Most importantly, the function $L(\omega)$ quantifies the degree of isotropy of the sampling scheme and thus provides some information on the quality of the directional rosette or Schmidt diagram synthesizing the results. If $L(\omega)$ remains approximately constant, the variations in directional density can be more safely analyzed than in the reverse situation.

This approach does not claim for originality: While reviewing the literature for the present paper, the authors found a very similar approach in Kiraly (1969), who himself referred to Muller (1963). The approach of Zhang and Einstein (2000), thought different, is also of the same vein. But many practical studies do not account correctly for the geometrical bias inherent to scanline surveys, or even ignore it.

EXTENSION TO AREAL SURVEYS

The above approach can be easily extended to an areal survey, where fracture traces are sampled on a planar outcrop. Let β denote the unit vector normal to the outcrop, S the outcrop surface, ℓ_i the length of that part of fracture trace i in the outcrop. By considering the outcrop as a volume with a very thin thickness e, the surface of fracture i within this volume is $\ell_i\, e \,/\, |\sin(\omega_i - \beta)|$, so that the equivalent to Terzaghi correction (4) in the case of several outcrops with surfaces summing to S would be

$$\hat{\mu}(d\omega) = \frac{1}{S}\sum_{i=1}^{N} \frac{\ell_i 1_{\omega_i \in d\omega}}{|\sin(\omega_i - \beta_i)|}$$

where β_i is the normal to the outcrop where fracture i has been measured. In the approach we propose, a station with surface S_{sta} orthogonal to the unit vector β provides neither more nor less information on fractures with pole ω than a station orthogonal to the fractures and with surface $S_{\text{sta}}\,|\sin(\omega - \beta)|$. This leads to define an equivalent surface, function of ω, as

$$S(\omega) = S_{\text{sta}}\,|\sin(\omega - \beta)|$$

When several stations are available with surfaces $S_1, \ldots, S_n$, and normals $\beta_1, \ldots, \beta_n$, respectively, the total equivalent surface for fractures with direction ω is

$$S(\omega) = S_1\,|\sin(\omega - \beta_1)| + \ldots + S_n\,|\sin(\omega - \beta_n)|$$

so that the equivalent to formula (5) is

$$\hat{\mu}(d\omega) = \frac{\sum_{i=1}^{N} \ell_i\, 1_{\omega_i \in d\omega}}{\sum_{j=1}^{n} S_j\,|\sin(\omega - \beta_j)|}$$

APPLICATION

The French National Agency for Radioactive Waste Management (or Andra) set up an underground research laboratory in the east part of France to study the feasibility of a deep geological waste repository in clay for high-level and long-lived intermediate-level radioactive waste. This laboratory is located in the eastern rim of the Paris Basin. In the studied zone, the Paris Basin consists of (from bottom to top): the Dogger limestone formation, with two units, the Bajocian and Bathonian units; the Callovo-Oxfordian argillite (selected host layer); and the Middle to Upper-Oxfordian limestone formation.

A statistical analysis of fracturing in Dogger and Oxfordian limestones has been carried out in order to make up the starting point for hydrological modelling. At the location of the laboratory the argillite layer is located from 422 up to 552 m in depth; the study was therefore done with data sampled at outcrops distant of several ten kilometres from the laboratory.

In the field, 23 sites displaying exposures of good quality were selected for scanline surveys along 20 to 80 m. A systematic scanline record of 1378 fractures of all types has been realised. Sub-vertical joints are by far the most abundant structures (87% of the measures).

Directional density has been analyzed by geological unit (Bajocian, Bathonian, Oxfordian), fracture type (fault, joint, vein, stylolitic joint), and importance class (major joints intersecting the whole height of the outcrop; joints crossing several layers; joints confined to a single layer). We focus on joints confined to a single layer, which is the most frequent class (864 fractures).

Since the joints are subvertical, the analysis of their orientation was carried out in terms of azimuth. The graphs of $L(\theta)$ and $\mu(\theta)$ show the following (Fig. 3):

- In the Oxfordian formation, even if the sampling scheme is very anisotropic and with a rather short equivalent length ($L(\theta)$ varies between 40 and 140 m), fracturation density brings out two major sets, centred on N45 and N135.
- The situation is similar in the Bathonian unit ($L(\theta)$ varies between 50 and 150 m), but the N135 set broadens with the emergence of a secondary set N0.
- In the Bajocian unit on the contrary, the graph of $\mu(\theta)$ as a function of fracture direction is rather chaotic and brings out no dominant set.

We could suspect this last observation to be due to a poor quality of the sampling in that geological unit. In fact this is not the case: the Bajocian unit has the largest number of stations (11). The good quality of the sampling in this unit is confirmed by the graph of $L(\theta)$ which is the richest and the most regular ($L(\theta)$ varies from 150 up to 210 m). These facts validate the above conclusions.

This was a surprise, because one expected a difference between the limestones below and above the argillite. In fact the Upper-Dogger and Oxfordian units encompassing the argillite layer behave similarly, whereas the main difference occurs between the Lower and Upper-Dogger units.

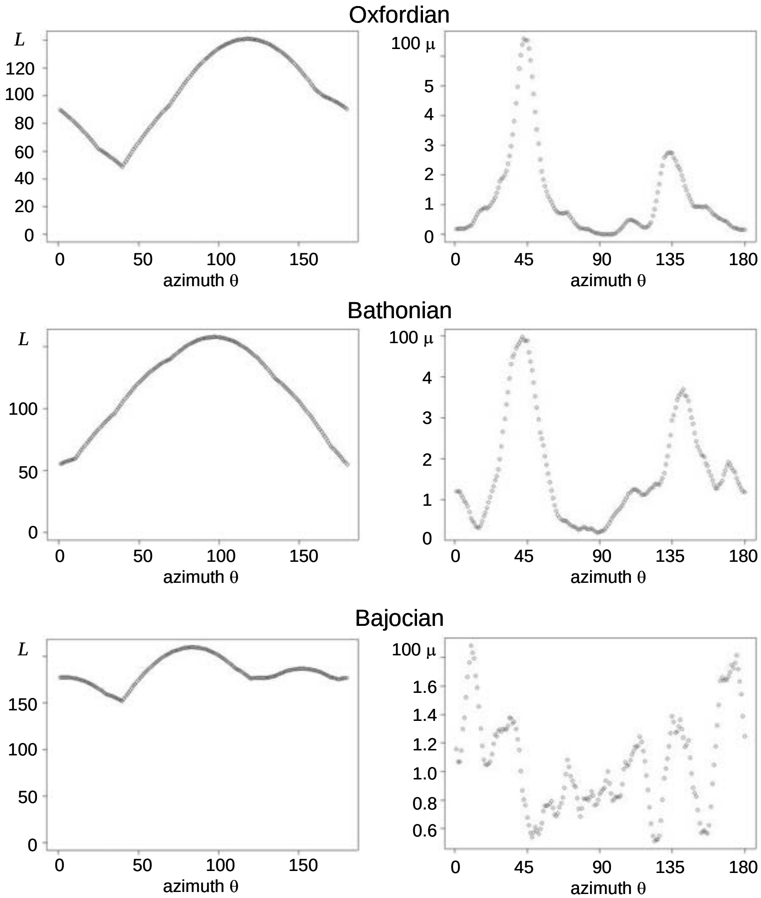

Figure 3. Graphs of equivalent length L (left) and fracture density μ (right) as functions of fracture direction θ, in the three geological units. Azimuth θ in degrees from North, L in metres, μ in m^2 per m^3 and per degree.

Table 1. Fracturation density of joints for each geological unit and each directional set.

Unit	D1 (N20-N60)	D2 (N110-N160)	D3 (N160-N20)	D4 (N60-N110)	Total
Bajocian	0.63	0.81	0.87	0.65	2.96
Bathonian	1.84	1.38	0.84	0.30	4.36
Oxfordian	1.50	0.87	0.21	0.14	2.72

These results as well as the examination of directional rosettes of the individual stations led to the definition of four directional joint sets, which correspond to the two major sets (D1 and D2) and two intermediate sets (D3 and D4). Table 1 gives their definition and their fracture density in each unit, obtained by integration of formula (5). We notice that the total fracture density has nearly the same magnitude in the three units, since it varies between 3 and 4 m^2/m^3, with the highest value in the Bathonian unit.

DISCUSSION AND CONCLUSION

The estimation of fracture density is a mere part of the study of a fractured medium but it may have a major role. For example, when the other parameters are fixed, the value of fracture density will decide whether the medium is below or above the percolation threshold, which has a major impact on the fluid flow regime.

The estimator of (directional) fracture density we propose improves Terzaghi correction. However, it does not exempt us from looking at the usual questions on the representativity of the data. Fracture density often varies spatially, so that the graph obtained from all the data is an average. As there is usually not much choice in the location of the stations, this average does not necessarily correctly represent the average in the study area. It is also necessary to model these spatial variations, if data permit. This is particularly important in layered formations, where the spacing of vertical joints confined to a single bed is often correlated with bed height. Lastly, extrapolating fracture parameters from outcrops to the underground remains an open question, because the decompression of rocks in the vicinity of the surface tends to develop new fractures.

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